Readington Township Public Schools

Grade 6 Pre-Algebra

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I. OVERVIEW

Grade 6 Pre-Algebra is a sixth grade math course designed to provide honors level students the opportunity to strengthen content knowledge in mathematics while accelerating the pace of instruction and promoting high levels of mathematical thought. Placement into this class is contingent upon meeting district criteria. The successful completion of this course will lead to the study of Algebra I in seventh grade.

This is a fast-paced course where students will cover the seventh grade Common Core State Standards (CCSS) as well as the eighth grade CCSS. Topics will include: Ratios and Proportional Relationships, The Number System, Expressions and Equations, Functions, Geometry, Statistics and Probability. Through their work in this course, students will understand and apply their knowledge in real world applications. Focus will be on the content as specified in the CCSS, as well as the CCSS Practice Standards. The Practice Standards focus on the development of competencies used by mathematicians in all grades and throughout life.

II. STUDENT OUTCOMES (Linked to the New Jersey Core Curriculum Standards/Common Core Mathematics Grade 7 and Common Core Mathematics Grade 8)

RATIOS AND PROPORTIONS (7.RP)

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

2. Recognize and represent proportional relationships between quantities.
    a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
    c. Represent proportional relationships by equations ($t = pn$).
    d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

THE NUMBER SYSTEM (7.NS)

Apply and extend previous understandings of operations with fractions.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
    a. Describe situations in which opposite quantities combine to make 0.
    b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
    c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
    d. Apply properties of operations as strategies to add and subtract rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
    a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
    b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then -(*p*/*q*) = (-*p*)/*q* = *p*/(-*q*). Interpret quotients of rational numbers by describing real-world contexts.
    c. Apply properties of operations as strategies to multiply and divide rational numbers.
    d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers (extend to complex fractions).

THE NUMBER SYSTEM (8.NS)
Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

EXPRESSIONS AND EQUATIONS (7.EE)
Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
1. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

2. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
    a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
    b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where *p*, *q*, and *r* are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

EXPRESSIONS AND EQUATIONS (8.EE)

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers).
    b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
    b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
    c. Solve real-world and mathematical problems leading to two linear equations in two variables.

FUNCTIONS (8.F)

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

GEOMETRY (7.G)

Draw construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

GEOMETRY (8.G)

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
    a. Lines are taken to lines, and line segments to line segments of the same length.
    b. Angles are taken to angles of the same measure.
    c. Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.

7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

STATISTICS AND PROBABILITY (7.SP)

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
    a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
    a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
    b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
    c. Design and use a simulation to generate frequencies for compound events.

STATISTICS AND PROBABILITY (8.SP)

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between two variables.

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

III. ESSENTIAL QUESTIONS AND CONTENT

Unit 1 Rational Numbers

- How are the properties of operations used to perform operations on rational numbers?
- How can the properties of integer exponents be used to generate equivalent numerical expressions?
- How are the four operations with rational numbers used to solve real-world and mathematical problems?

Unit 2 Proportional Relationships

- How can rates be used to describe real-world problems?
- How can proportional reasoning be used to solve real-world problems?
- How can a unit rate or constant of proportionality be identified in a table, graph, or equation?

Unit 3 Linear Relationships

- How can the properties of operations be used to simplify linear expressions?
- How are the coordinates of a point on a line or in a table related to the equation of the line?
- How can equations with variables on both sides be solved?
- What are the characteristics of a linear equation, and how is a linear equation graph constructed using a table of values or linear equations?

Unit 4 Statistics and Probability

- Why is data collected, and how do I work with and represent data in meaningful ways?
- How can the slope and intercept be interpreted in the equation of a linear model to solve problems in the context of bivariate measurement data?
- How can the experimental probability of a chance event be determined?
- How can a probability model be developed and used to find probabilities of events?
- What are possible sources of discrepancy when comparing probabilities from a model to observed frequencies if the agreement is not good?
- How can organized lists, tables, tree diagrams, and simulations be used to find probabilities of compound events?

Unit 5 Geometry

- What are the properties and relationships of angles when they are found in polygons, adjacent to each other, formed by intersecting lines, or created by transversals?
- What formulas can I use to solve real-world problems involving shapes, area, and volume?
- Can I use ratio and proportion concepts to change the scale of a drawing?
- How can I create shapes with specific criteria?
- How do I use the many special properties of triangles to describe them?
- Why are roots and irrational numbers important to geometry?
- How can the Pythagorean Theorem be used to solve real-world and mathematical problems in two and three dimensions?

IV. STRATEGIES

The curriculum will be instructed through a variety of strategies based in research on middle school learning and educational best practices. Students will be engaged in meaningful lessons and activities using guided and independent practice, cooperative learning, and discovery in order to develop cognitive skills intended to promote independence of thought and analysis. Students will participate in hands-on activities, use manipulatives where appropriate and participate actively in class discussions.

Teachers will encourage students to employ a number of problem solving strategies relevant to the situation. Students will demonstrate evidence of understanding through modeling, written explanations, verbal descriptions and oral presentations. Students may also use tools of technology where needed to better enhance their ability to complete and defend their mathematical reasoning.

V. EVALUATION

- Teacher observations of student performance in class will be used as formative assessment.
- Homework assignments will primarily be used as practice, and reinforcement of concepts and skills.
- Notebooks may be used as formative assessment of student understanding.

- Student projects
- Unit tests and quizzes will be given and graded based on a total points system, consistent with the grading principles at the middle school.
- Benchmark unit assessments

VI. REQUIRED RESOURCES

Big Ideas Math Course 2 Accelerated by Ron Larson and Laurie Boswell; Big Ideas Learning and associated *Record and Practice Journal* along with other teacher selected resources.

The following resources can also be used as reference, or may be used by the course instructor.

- Grade 7 Common Core Unpacked Standards Document
- Grade 8 Common Core Unpacked Standards Document
- Project Based Assignment Resources – Including, but not limited to:
  - Illustrative Mathematics (www.illustratviemathematics.org)
  - The MAP Project (www.map.mathshell.org/materials/index.php)

VII. SCOPE AND SEQUENCE

Unit 1 Rational Numbers

Integers and Rational Numbers (4 weeks)

1. Extending the Number System
2. Adding and Subtracting Integers
3. Multiplying and Dividing Integers
4. Properties of Operations
5. Properties of Integer Exponents
6. Scientific Notation and Operations
7. Adding and Subtracting Rational Numbers
8. Multiplying and Dividing Rational Numbers
9. Terminating and Repeating Decimals
10. Problem Solving with Rational Numbers

Unit 2 Proportional Reasoning

Ratios and Proportions (4 weeks)

1. Ratios and Proportions
2. Rates and Unit Rates
3. Comparing and Scaling Rates
4. Writing and Solving Proportions
5. Unit Rate and Constant of Proportionality
6. Graphing Proportional Relationships

Percent (3 weeks)

1. The Percent Proportion
2. The Percent Equation
3. Percent of Increase and Decrease
4. Discounts and Tax
5. Markups and Markdowns
6. Gratuities and Commissions
7. Simple Interest
8. Percent Error

Unit 3 Linear Relationships

Walking Rates (1 week)

1. Finding and Using Rates
2. Tables, Graphs, and Equations
3. Linear Patterns of Change
4. Recognizing Linear Relationships

Exploring Linear Relationships with Graphs and Tables (1 week)

1. Finding the Point of Intersection
2. Using Tables, Graphs, and Equations
3. Comparing Relationships
4. Connecting Tables, Graphs, and Equations

Expressions and Equations (3 weeks)

1. Simplifying Linear Expressions
2. Adding and Subtracting Linear Expressions
3. Factoring and Expanding Linear Expressions
4. Solving Equations Using Tables and Graphs
5. Exploring Equality
6. Writing Equations
7. Solving Linear Equations

Inequalities (1 week)

1. Writing and Graphing Inequalities
2. Solving Inequalities Using Addition or Subtraction
3. Solving Inequalities Using Multiplication or Division
4. Solving Two-Step Inequalities

Exploring Slope (2 weeks)

1. Using Rise and Run
2. Finding the Slope of a Line
3. Similar Triangles
4. Exploring Patterns With Lines (parallel and perpendicular)
5. Comparing Linear Relationships Represented in Different Ways

Graphing (1 week)

1. Graphing Linear Equations
2. Graphing Proportional Relationships
3. Slope-Intercept Form

Unit 4 Statistics and Probability

Samples and Populations (2 weeks)

1. Representative Samples
2. Random Samples
3. Comparing Populations
4. Making Predictions
5. Measures of Center and Measures of Variability

Bivariate Data (1 week)

1. Scatter plots
2. Patterns of Association including clustering and outliers

3. Line of Best Fit
4. Using Linear Models to Solve Problems

Probability (4 weeks)
1. Experimental Probabilities
2. Equally Likely Events
3. Theoretical Probabilities
4. Probability Models
5. Sample Space
6. Tree Diagrams
7. Compound Events
8. Making Predictions
9. Designing and Using a Simulation
10. Expected Value
11. Independent and Dependent Events

Unit 5 Geometry

Angle Measure, Constructions and Scale Drawings (2 weeks)
1. Adjacent and Vertical Angles
2. Complementary and Supplementary Angles
3. Parallel Lines and Transversals
4. Angles of Triangles
5. Angles of Polygons
6. Constructing Triangles
7. Scale Drawings

Area, Surface Area, and Volume (4 weeks)
1. Circumference of Circles
2. Areas of Circles
3. Areas of Composite Figures
4. Surface Area of Prisms
5. Surface Area of Pyramids
6. Surface Area of Cylinders
7. Volume of Prisms
8. Volume of Pyramids
9. Volume of Cylinders
10. Volume of Cones
11. Volume of Spheres
12. Solve Problems Involving Area, Surface Area, and Volume

Transformations (1 week)
1. Congruent Figures
2. Translations
3. Reflections
4. Rotations
5. Similar Figures
6. Dilations

The Pythagorean Theorem (2 weeks)
1. Finding Square Roots
2. The Pythagorean Theorem

3. Approximating Square Roots
4. Using the Pythagorean Theorem

Unit 6: Project Based Learning (2 weeks)

Students will complete a number of projects to review and/or extend topics covered in this course. Projects will vary in duration and form, and will be based on real-world situations and examples. Students will be required to apply and extend learning through their responses, calculations and/or presentations.